

Temperature Multi-stability

From non-linear radiation terms

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CriticalEarth



Key Points



The Model



Modifications



Results

Key Points



The Model

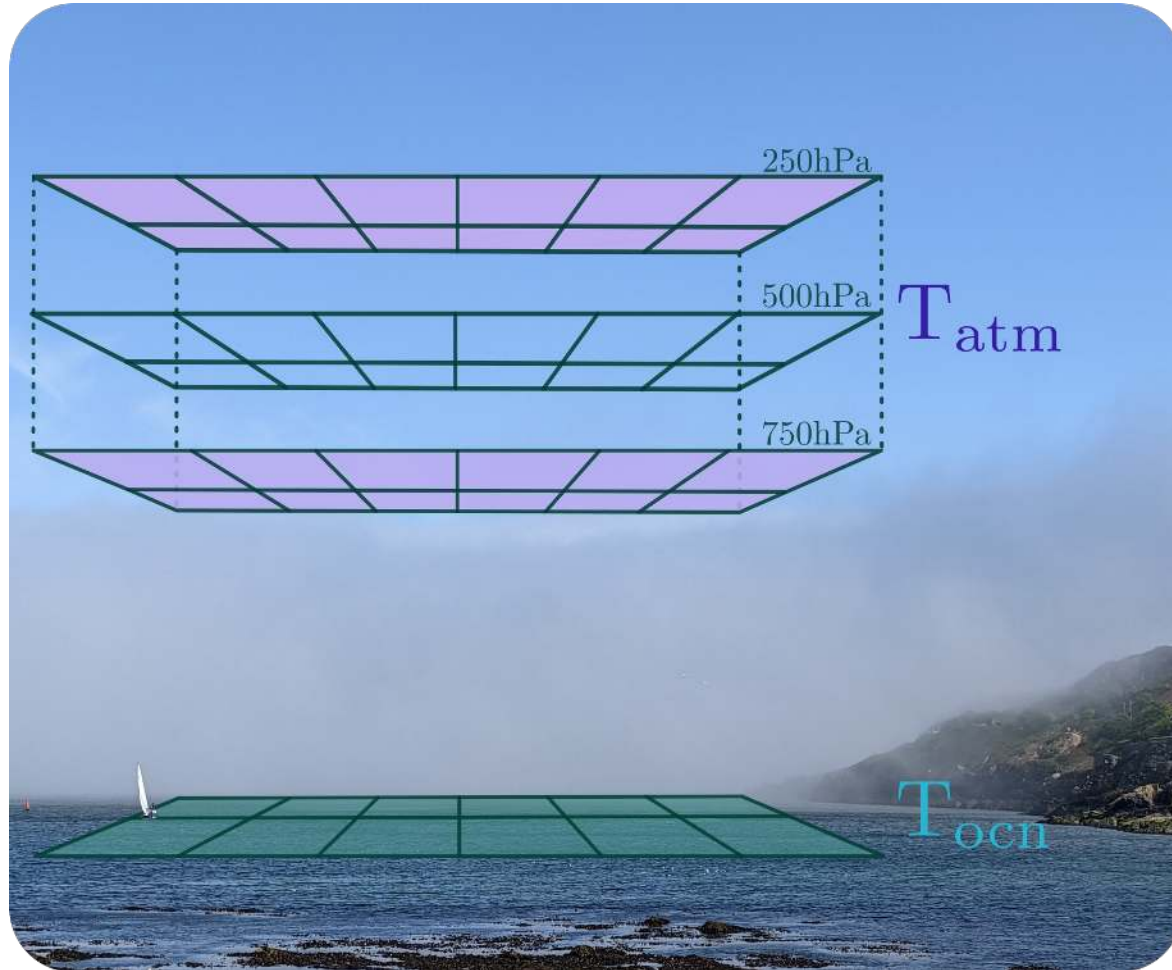
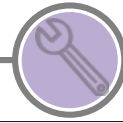


Modifications



Results

QGS Model

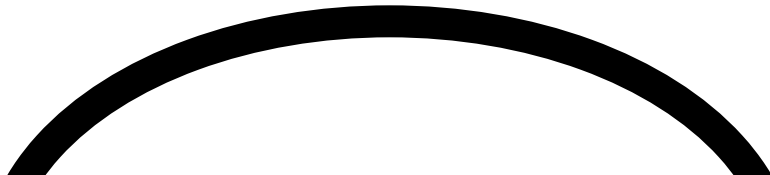


QGS Model

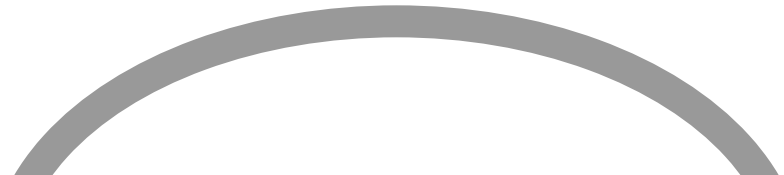
Stefan Boltzmann Law



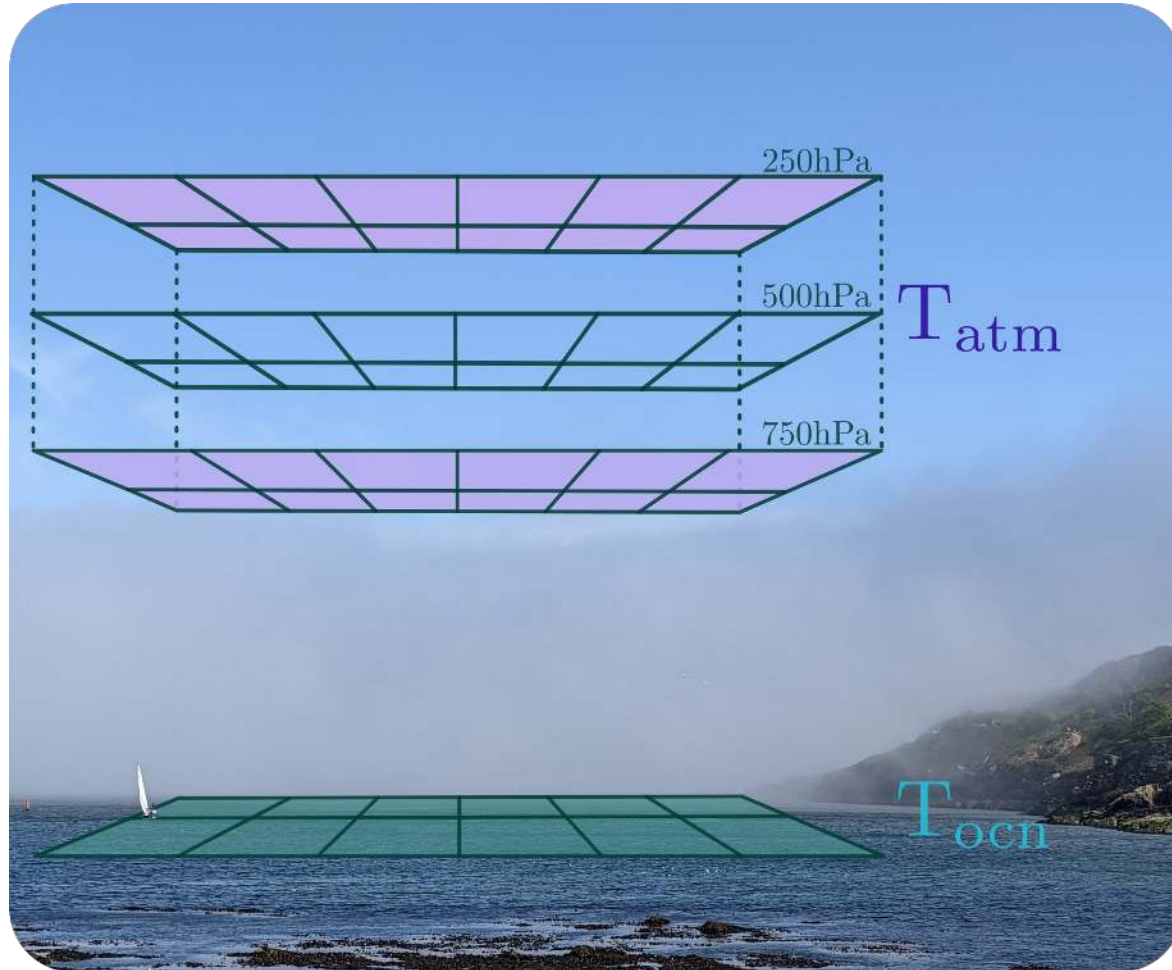
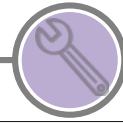
$$\sigma T^4$$



$$\epsilon \sigma T^4$$



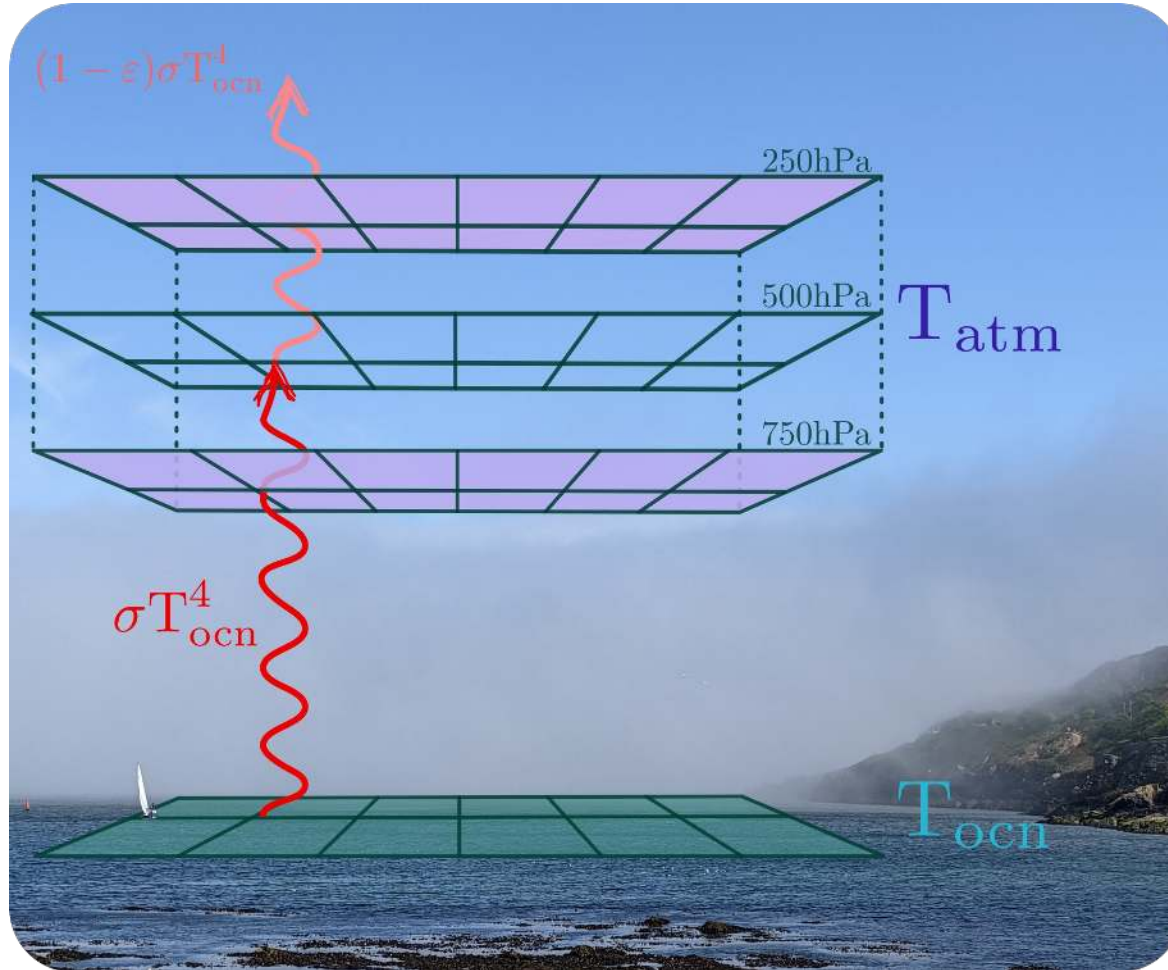
QGS Model



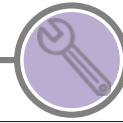
QGS Model



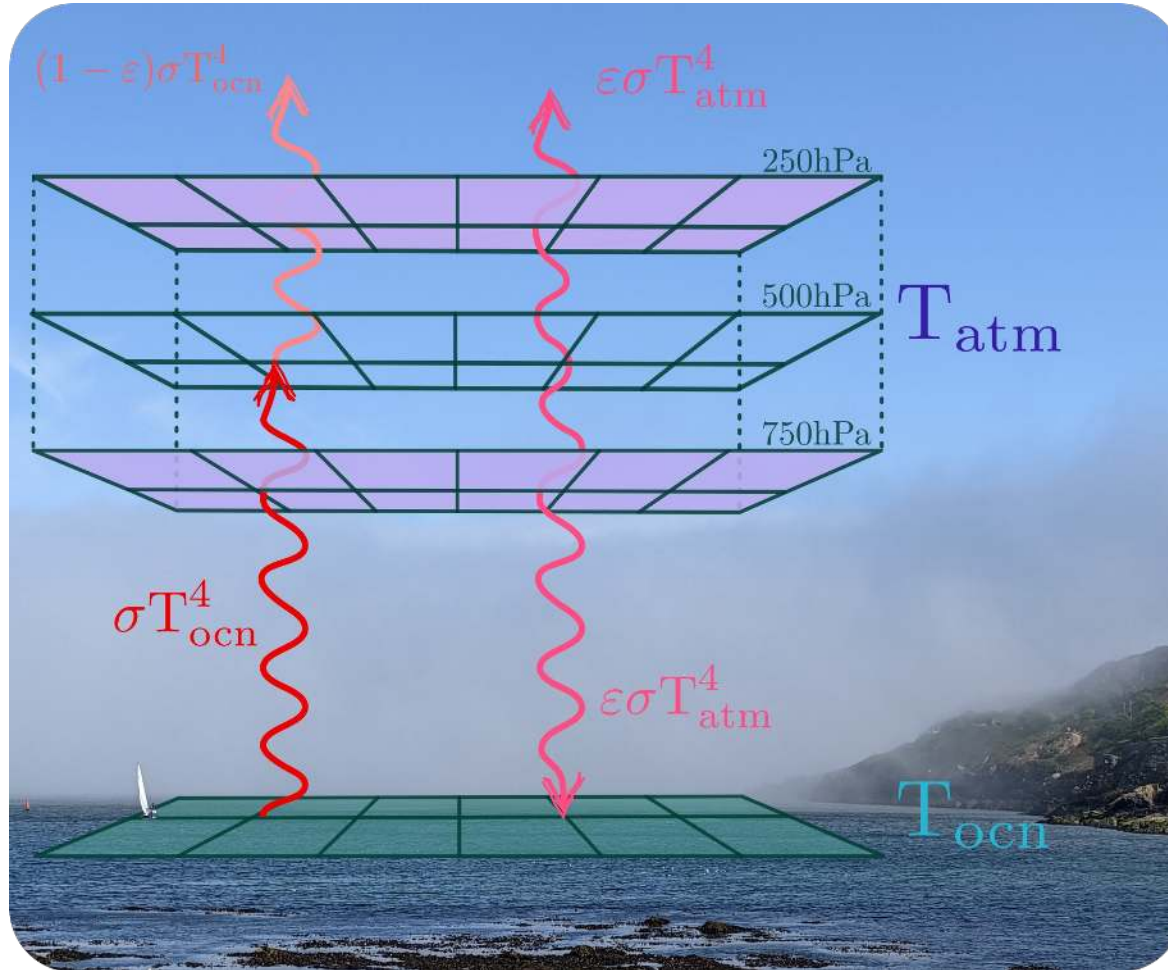
Radiation



QGS Model



Radiation



QGS Model



Temperature Equation

$$\gamma_o \left(\frac{\partial T_o}{\partial t} + J(\psi_o, T_o) \right) = -\lambda(T_o - T_a) - \sigma_B T_o^4 + \varepsilon \sigma_B T_a^4 + R_o$$

QGS Model



Temperature Equation

$$\gamma_o \left(\frac{\partial T_o}{\partial t} + J(\psi_o, T_o) \right) = -\lambda(T_o - T_a) - \sigma_B T_o^4 + \varepsilon \sigma_B T_a^4 + R_o$$

The equation is annotated with several symbols below it:

- A blue double-headed arrow is positioned below the term $\gamma_o \left(\frac{\partial T_o}{\partial t} + J(\psi_o, T_o) \right)$.
- An orange circular arrow is positioned below the term $-\lambda(T_o - T_a)$.
- A pink wavy arrow pointing upwards is positioned below the term $-\sigma_B T_o^4$.
- A pink wavy arrow pointing downwards is positioned below the term $+\varepsilon \sigma_B T_a^4$.
- A yellow wavy arrow pointing downwards is positioned below the term $+ R_o$.

QGS Model



Temperature Equation

$$\gamma_o \left(\frac{\partial T_o}{\partial t} + J(\psi_o, T_o) \right) = -\lambda(T_o - T_a) - \sigma_B T_o^4 + \varepsilon \sigma_B T_a^4 + R_o$$

The equation is annotated with several symbols below it: a double-headed horizontal arrow under the left-hand side; a circular refresh icon under the $-\lambda(T_o - T_a)$ term; a wavy arrow pointing up from a horizontal line under the $-\sigma_B T_o^4$ term; a wavy arrow pointing down from a horizontal line under the $+\varepsilon \sigma_B T_a^4$ term; and a wavy arrow pointing down from a horizontal line under the $+R_o$ term.

QGS Model



Linearisation

$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$

QGS Model



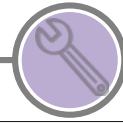
Linearisation

$$T_o = T_{o,0} + \delta T_o(t, x, y)$$

↓

$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$

QGS Model



Linearisation

$$T_o = \boxed{T_{o,0}} + \delta T_o(t, x, y)$$

$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$

$O(\delta T_o)$

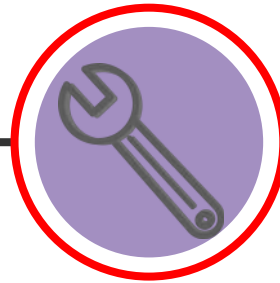
$$-4\sigma_B T_{o,0}^3 \delta T_o + 4\varepsilon \sigma_B T_{a,0}^3 \delta T_a$$



Key Points



The Model



Modifications

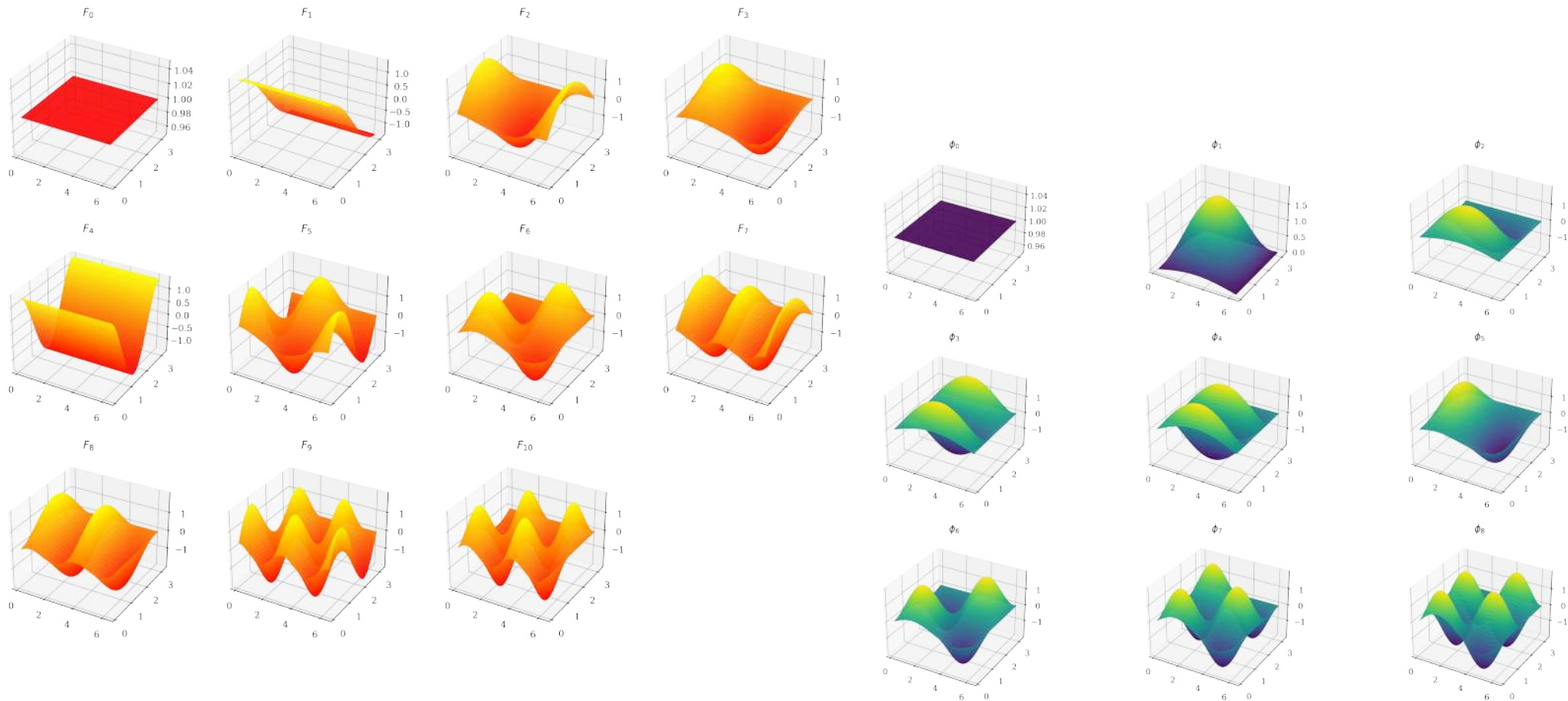


Results

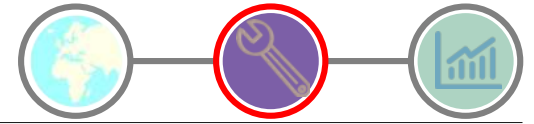
Modifications



New Modes



Modifications



Dynamic Equilibrium

$$T_o = T_{o,0}(t) + \delta T_o(t, x, y)$$

↓

$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$

Modifications



Dynamic Equilibrium

$$T_o = T_{o,0}(t) + \delta T_o(t, x, y)$$

$O(\delta T_o)$

$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$
A large grey arrow originates from the $\delta T_o(t, x, y)$ term in the equation above and points down to the $-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$ expression. A second large grey arrow originates from the $O(\delta T_o)$ label and points down and to the left.

Modifications



Dynamic Equilibrium

$$T_o = \boxed{T_{o,0}(t)} + \delta T_o(t, x, y)$$

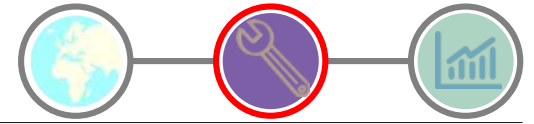
$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$

$O(\delta T_o)$

$$-\sigma_B T_{o,0}^4 + \varepsilon \sigma_B T_{a,0}^4 + f(\delta T_o) + g(\delta T_a)$$



Modifications



Non-Linear Equation

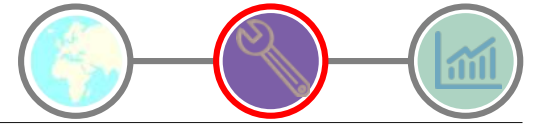
$$T_o(t, x, y)$$



$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$



Modifications



Non-Linear Equation

$$T_o(t, x, y)$$



$$-\sigma_B T_o^4 + \varepsilon \sigma_B T_a^4$$

Problem:




5-6x run time



Modifications

Model Summary



	Model	T^4 Radiation Terms	T_0 Equilibrium Temperature
	Linearised	Linearised	Constant
	Dynamic Temperature	Linearised	Dynamic
	Non-Linear	Non-Linearised	Dynamic

Key Points



The Model



Modifications

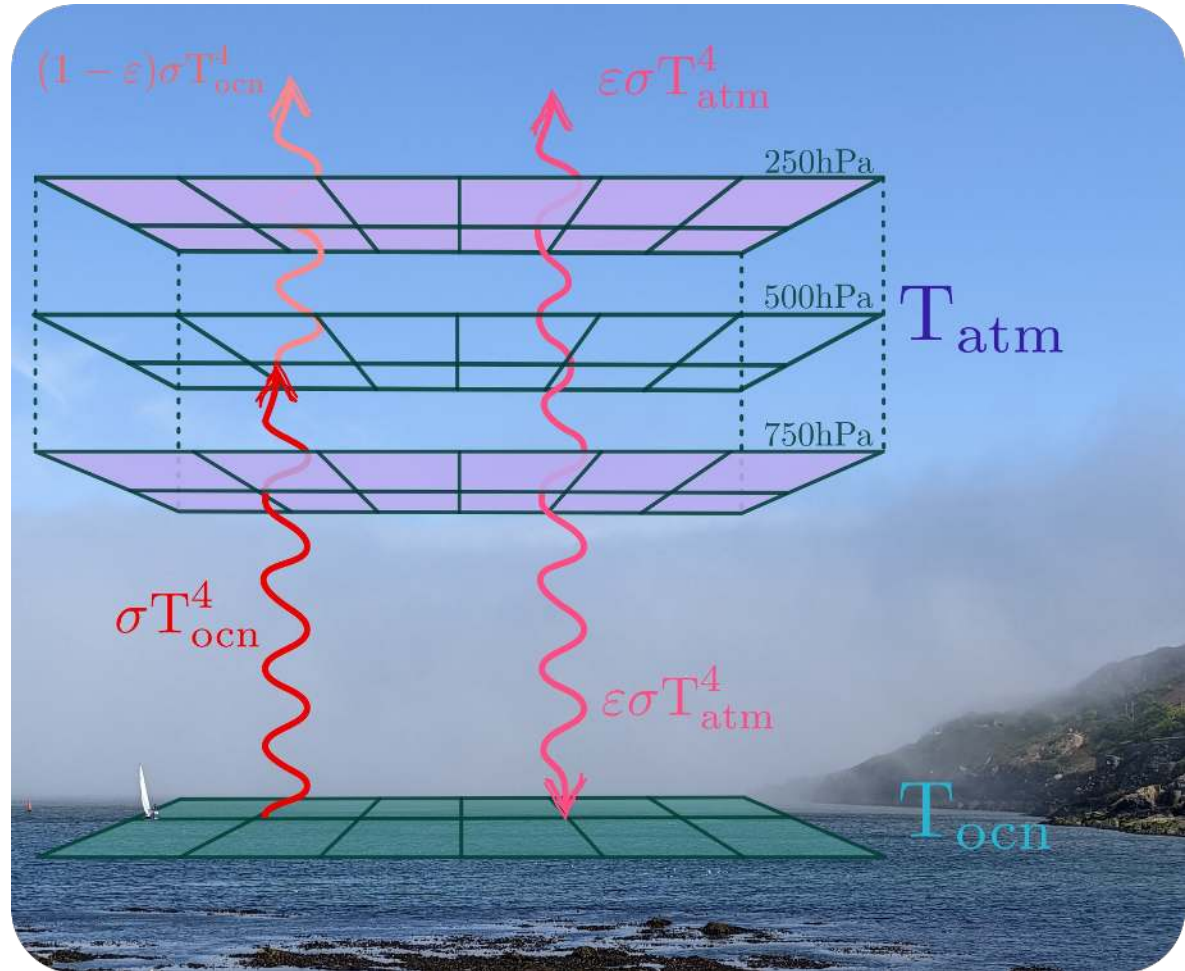


Results

Model Parameters



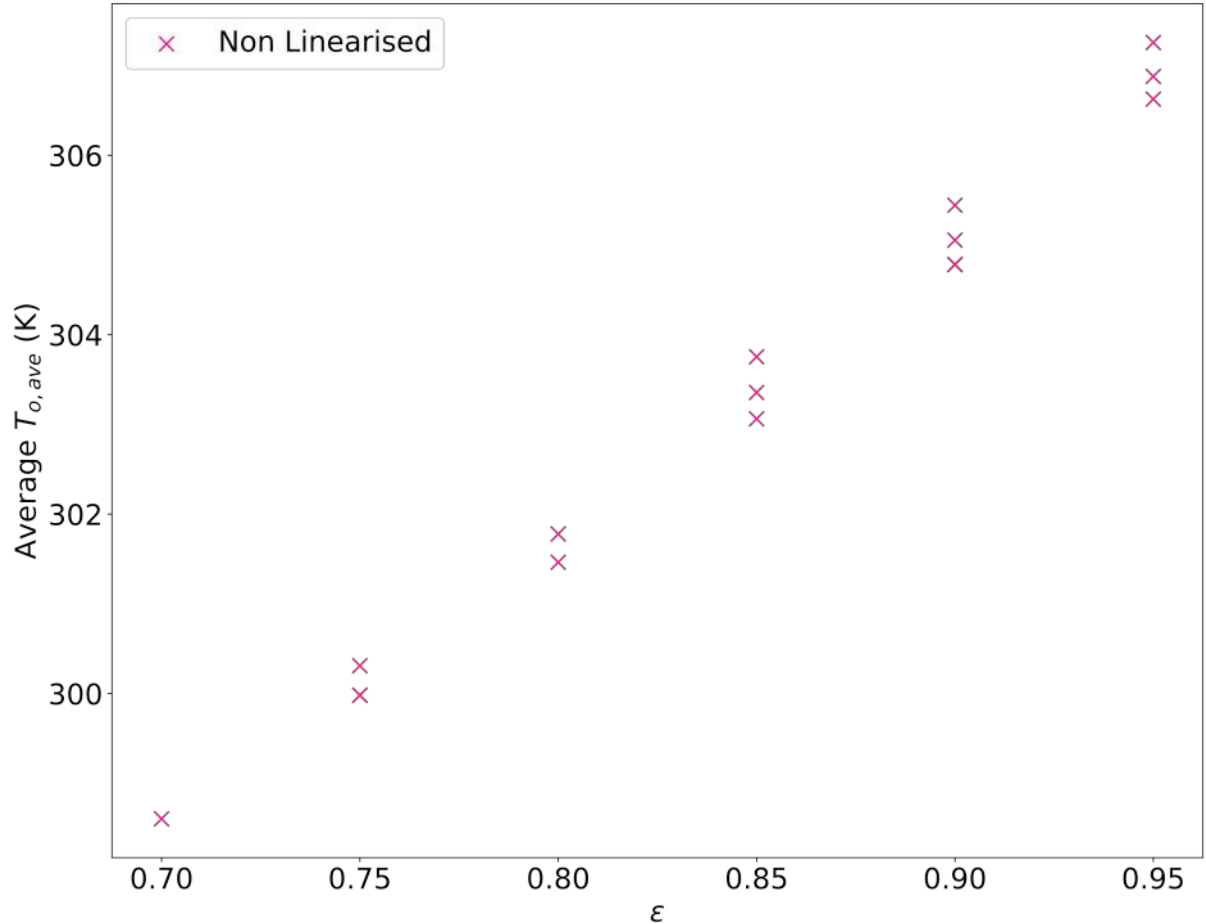
Emissivity ε



Model Parameters



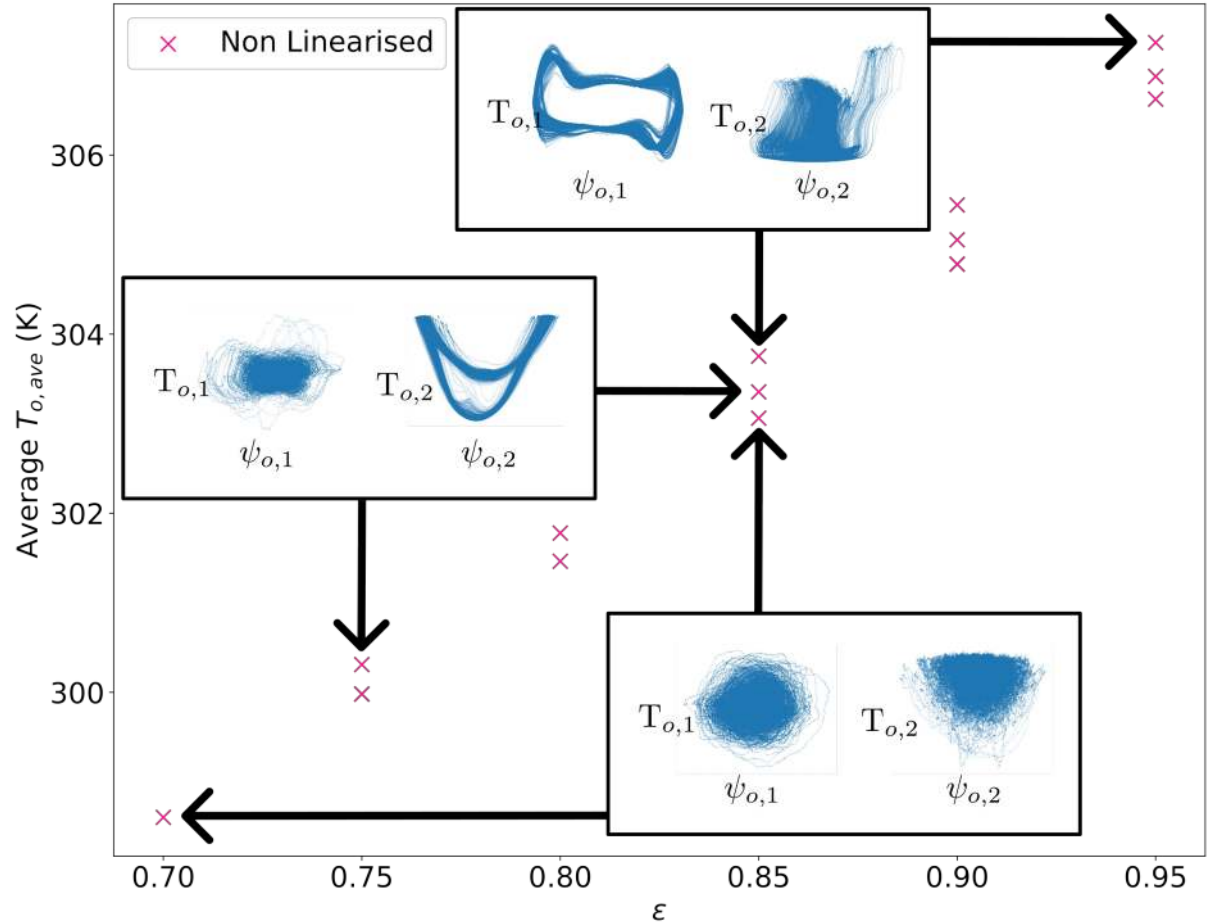
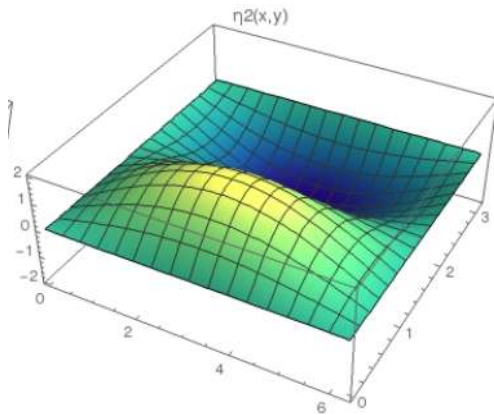
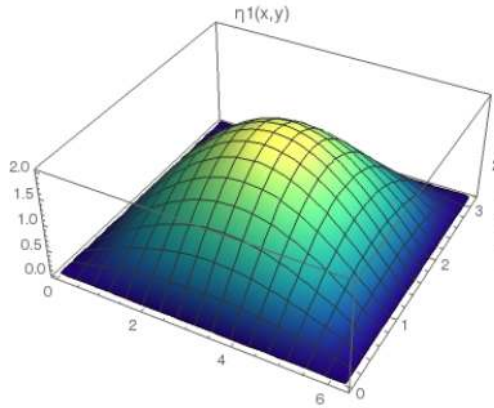
Emissivity ϵ



Model Parameters



Emissivity ε

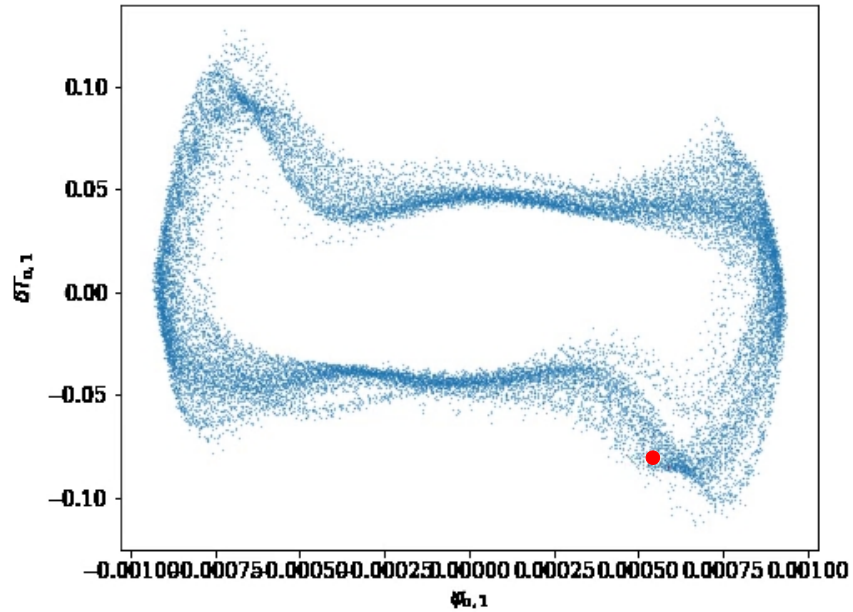


Model Parameters

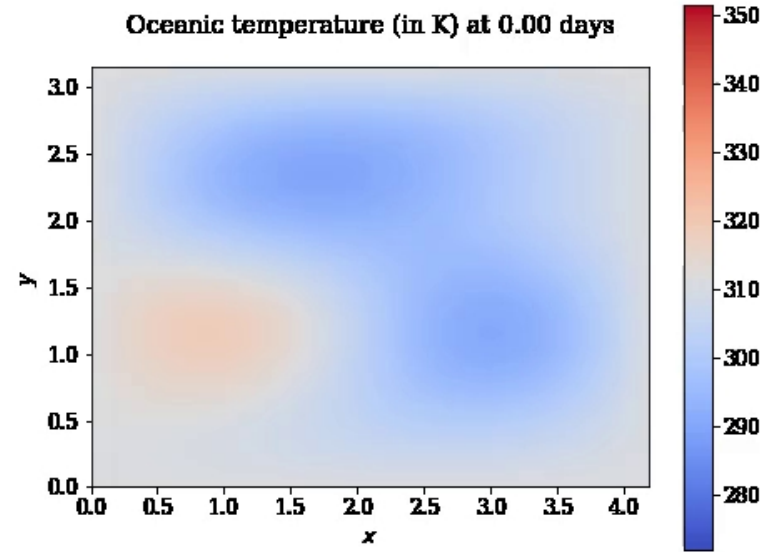


Emissivity ε

Model's variables (in nondim units)



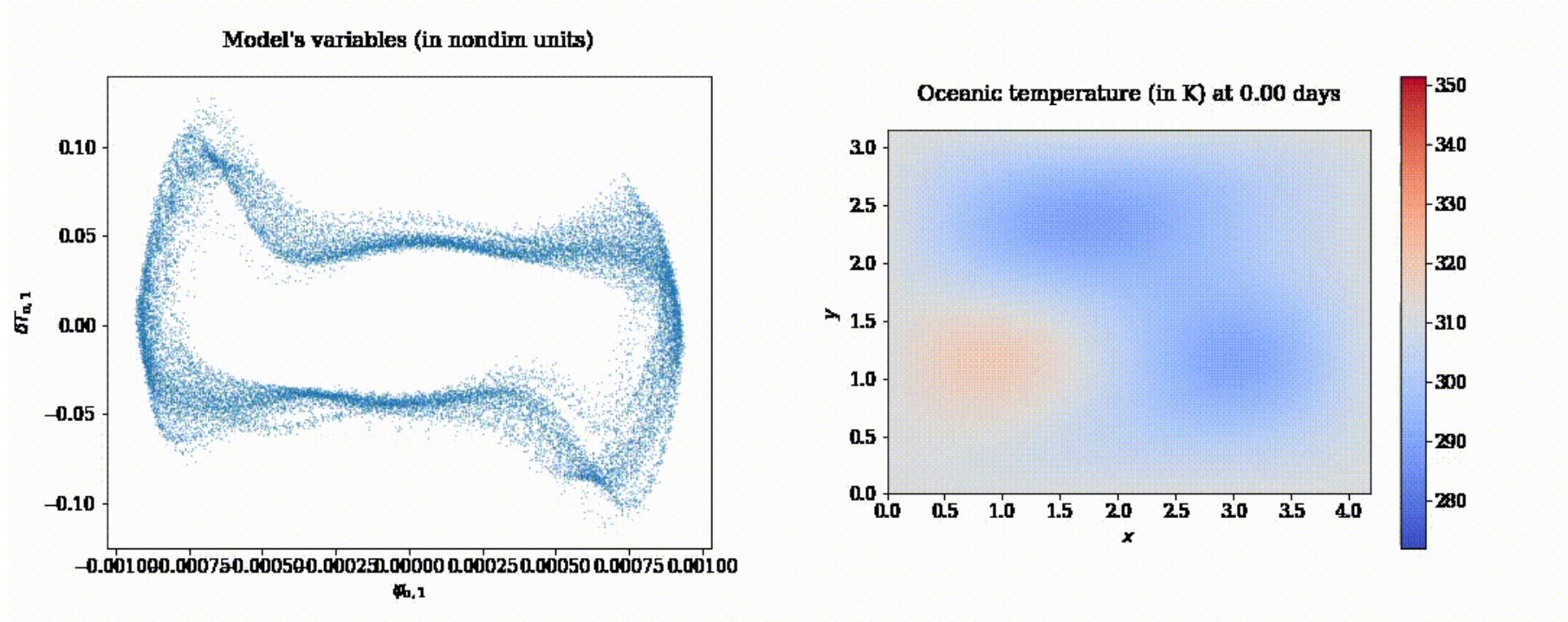
Oceanic temperature (in K) at 0.00 days



Model Parameters



Emissivity ε

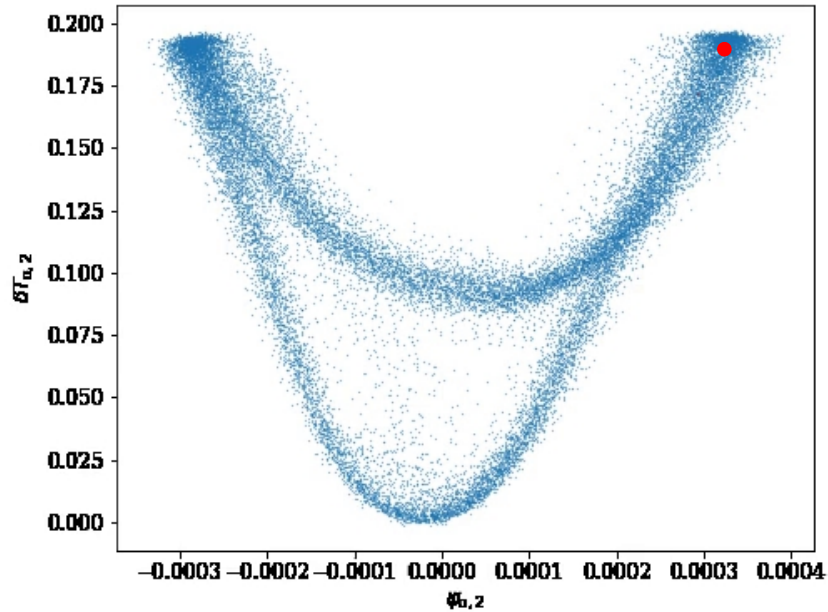


Model Parameters

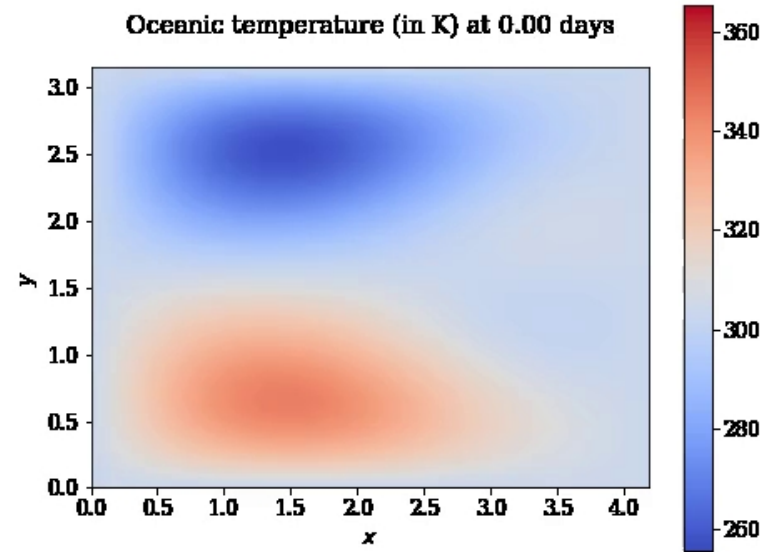


Emissivity ε

Model's variables (in nondim units)



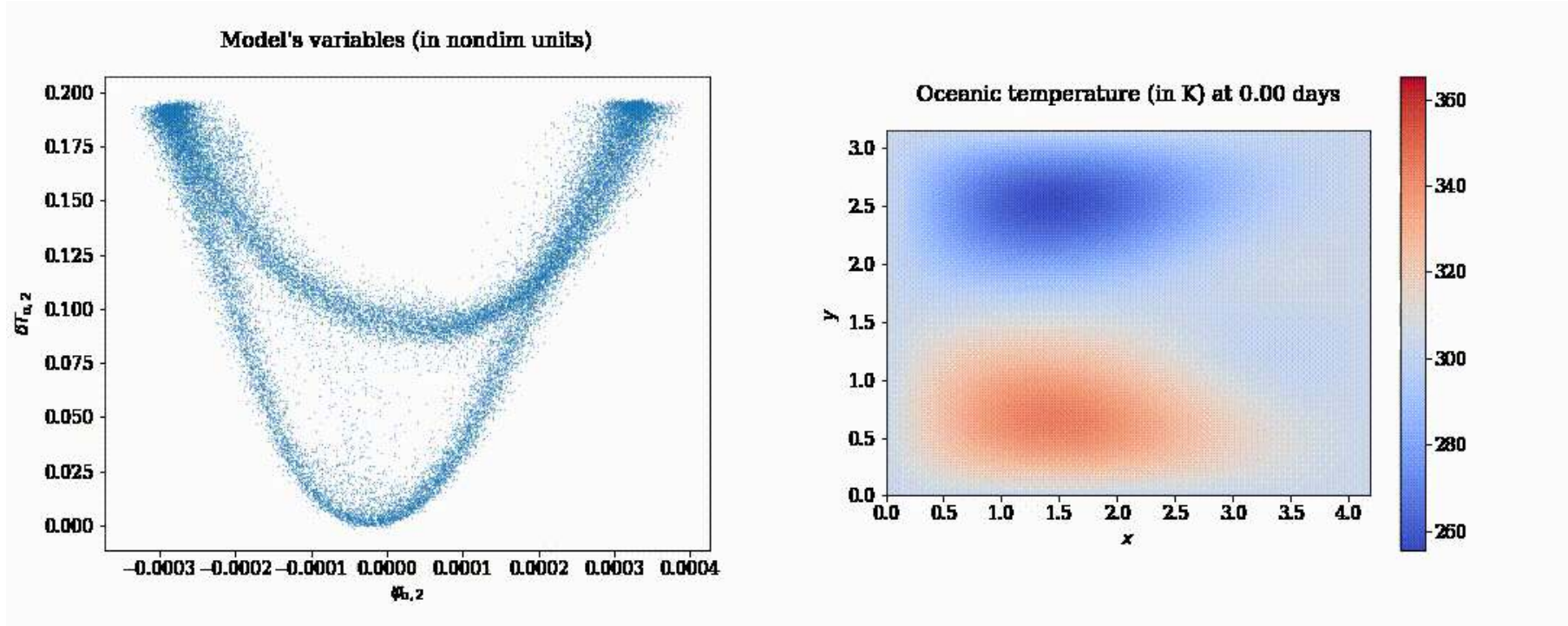
Oceanic temperature (in K) at 0.00 days



Model Parameters



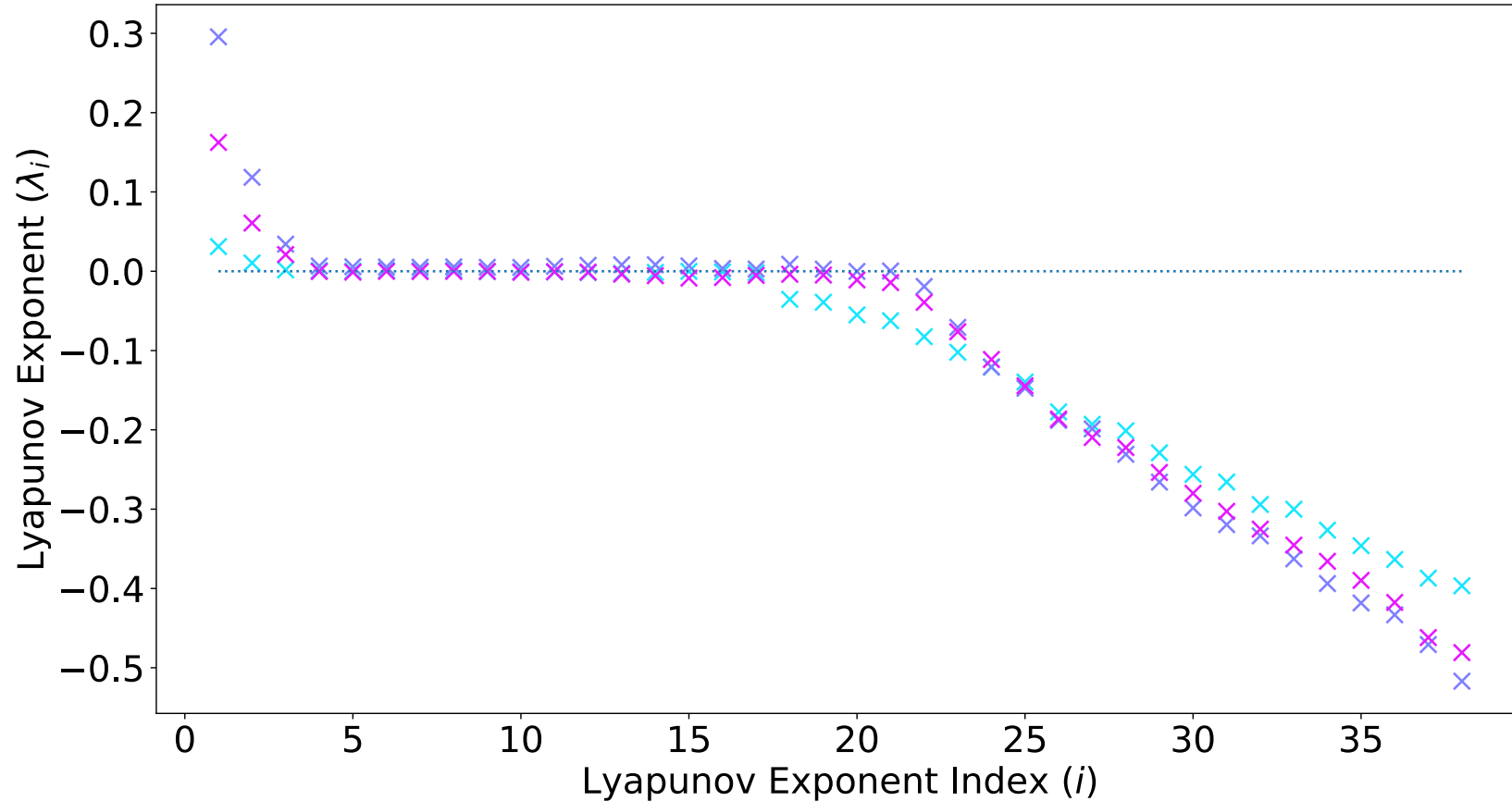
Emissivity ϵ



Model Outputs

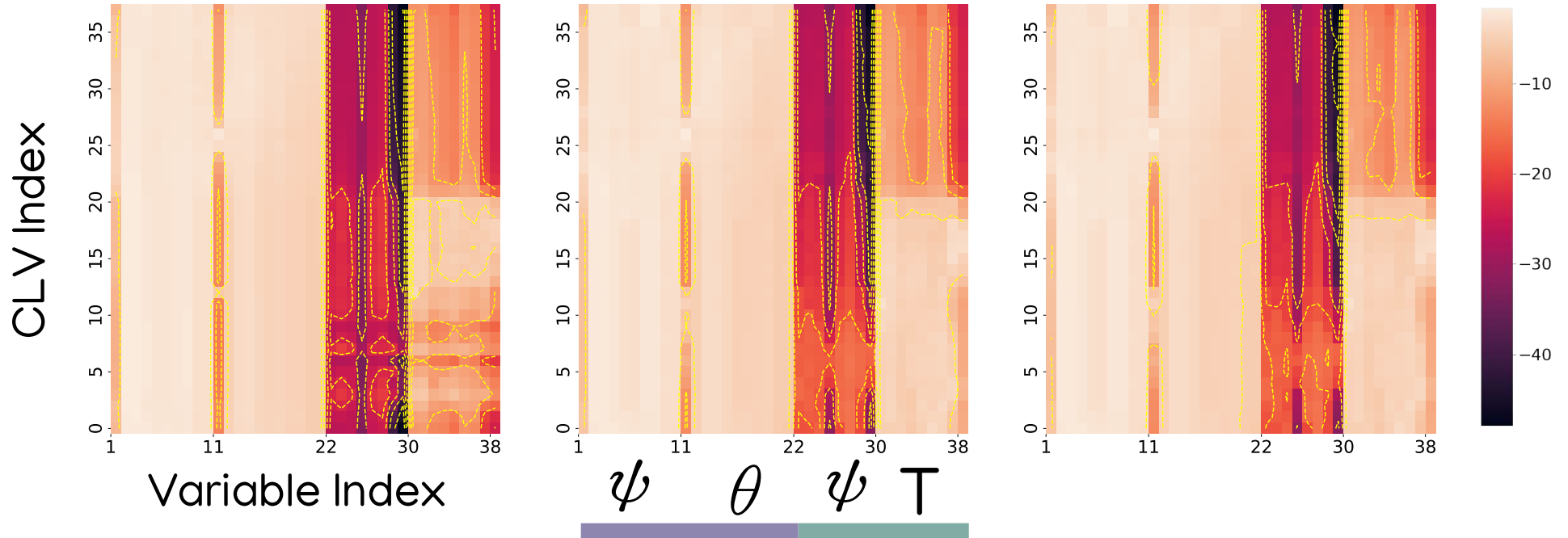


Lyapunov Exponents



Model Outputs

Lyapunov Exponents



Conclusion



Non linear radiation terms produce temperature multi-stabilities

Conclusion



Non linear radiation terms produce temperature multi-stabilities

Multi-stabilities produce distinct behaviour

Conclusion



Non linear radiation terms produce temperature multi-stabilities

Multi-stabilities produce distinct behaviour

Multi-stabilities in majority of cases produced by dynamic equilibria

Thank you

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De Cruz et al. (2016)
The Modular Arbitrary-Order
Ocean-Atmosphere Model:
MAOOAM v1.0