Multistability in a Coupled Ocean-Atmosphere Low Order Model

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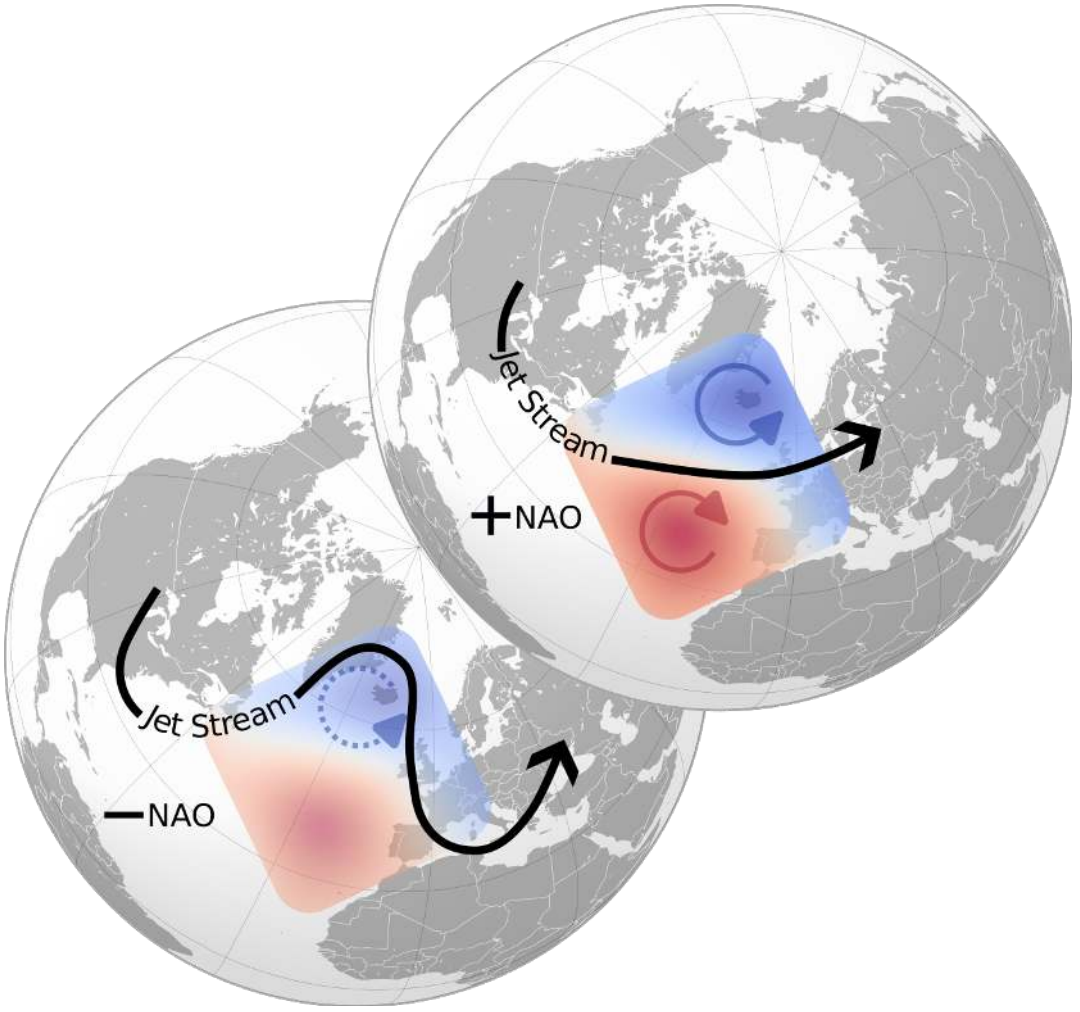
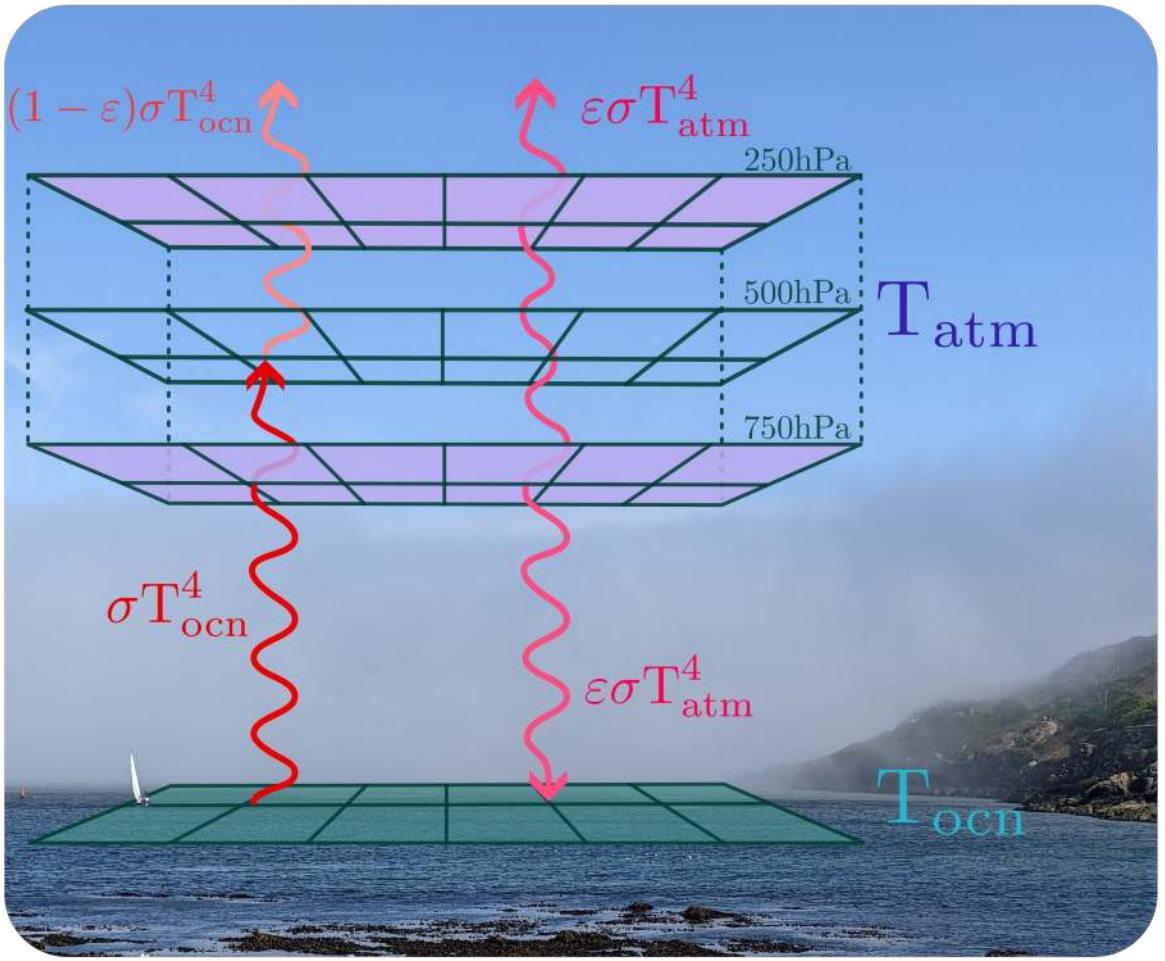
Motivation



qgs Model

Quasi-Geostrophic
Spectral model



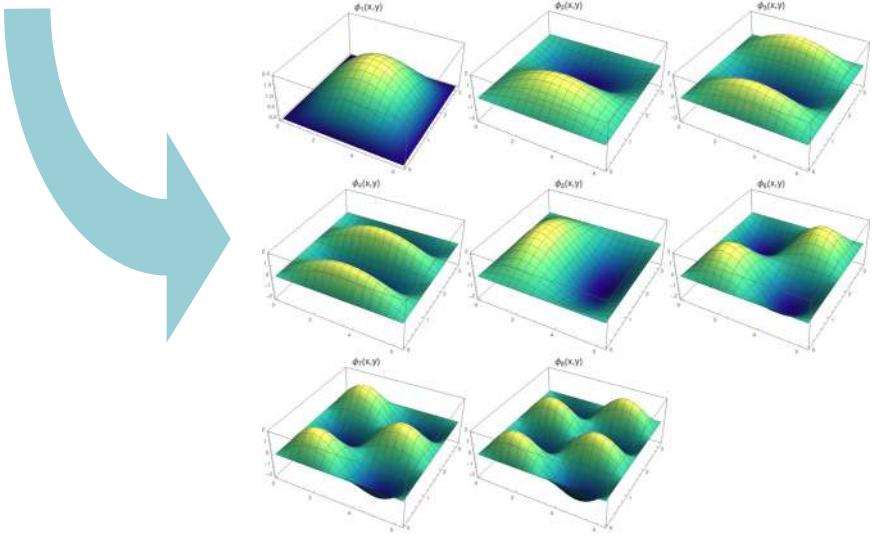
qgs Model

$$\frac{\partial}{\partial t}\left(\nabla^2\psi_\mathrm{a}\right) + J\left(\psi_\mathrm{a}, \nabla^2\psi_\mathrm{a}\right) + \beta\frac{\partial\psi_\mathrm{a}}{\partial x} = +k_d'\nabla^2\left(\psi_\mathrm{a}^1 - \psi_\mathrm{a}\right) - \frac{f_0}{\Delta p}\omega - k_d\nabla^2\left(\psi_\mathrm{a} - \psi_\mathrm{o}\right)$$

$$\frac{\partial}{\partial t}\left(\nabla^2\psi_\mathrm{o} - \frac{\psi_\mathrm{o}}{L_\mathrm{R}^2}\right) + J\left(\psi_\mathrm{o}, \nabla^2\psi_\mathrm{o}\right) + \beta\frac{\partial\psi_\mathrm{o}}{\partial x} = -r\nabla^2\psi_\mathrm{o} + \frac{C}{\rho_\mathrm{o}h}\nabla^2\left(\psi_\mathrm{a}^3 - \psi_\mathrm{o}\right).$$

$$\gamma_\mathrm{a}\left(\frac{\partial T_\mathrm{a}}{\partial t} + J\left(\psi_\mathrm{a}, T_\mathrm{a}\right) - \sigma\omega\frac{p}{R}\right) = -\lambda\left(T_\mathrm{a} - T_\mathrm{o}\right) + \epsilon_\mathrm{a}\sigma_\mathrm{B}T_\mathrm{o}^4 - 2\epsilon_\mathrm{a}\sigma_\mathrm{B}T_\mathrm{a}^4 + R_\mathrm{a}$$

$$\gamma_\mathrm{o}\left(\frac{\partial T_\mathrm{o}}{\partial t} + J\left(\psi_\mathrm{o}, T_\mathrm{o}\right)\right) = -\lambda\left(T_\mathrm{o} - T_\mathrm{a}\right) - \sigma_\mathrm{B}T_\mathrm{o}^4 + \epsilon_\mathrm{a}\sigma_\mathrm{B}T_\mathrm{a}^4 + R_\mathrm{o}$$



$$\dot{\psi}_{\mathrm{a},i} = \dots$$

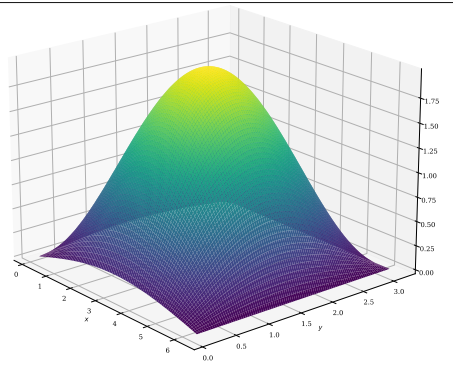
$$\dot{\theta}_{\mathrm{a},i} = \dots$$

$$\dot{\psi}_{\mathrm{o},i} = \dots$$

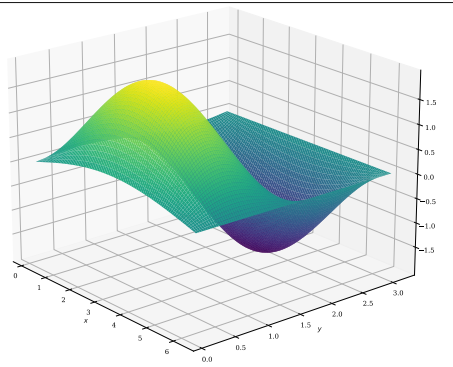
$$\dot{\delta T}_{\mathrm{o},i} = \dots$$

Model Results

ϕ_1

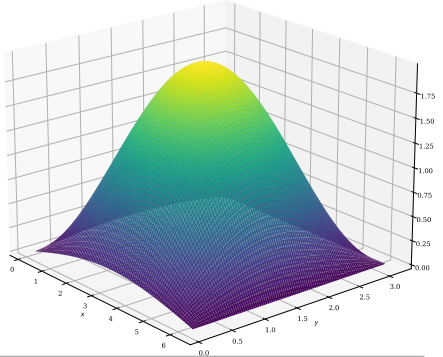


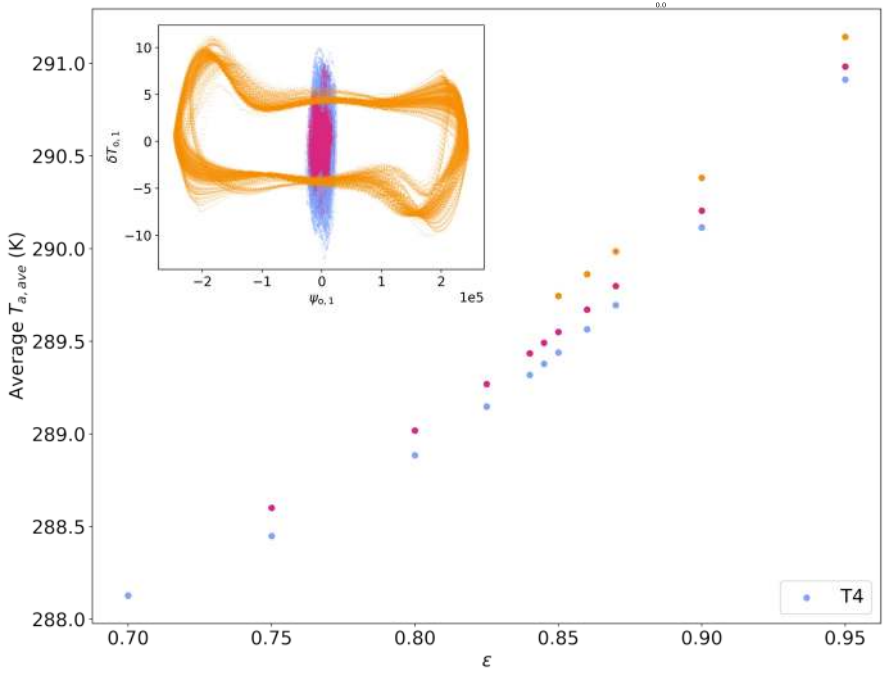
ϕ_2



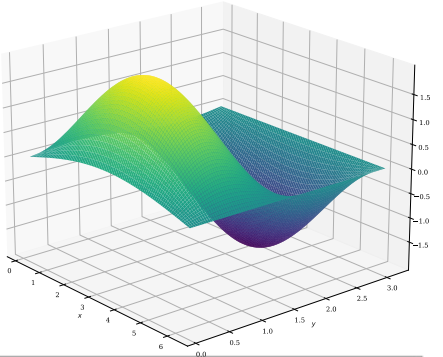
Model Results

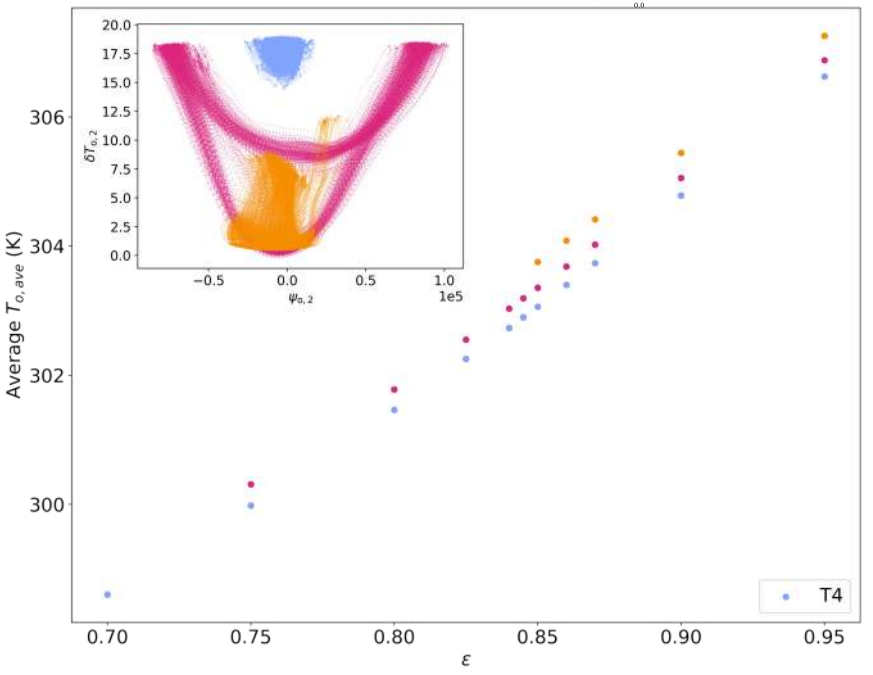
ϕ_1



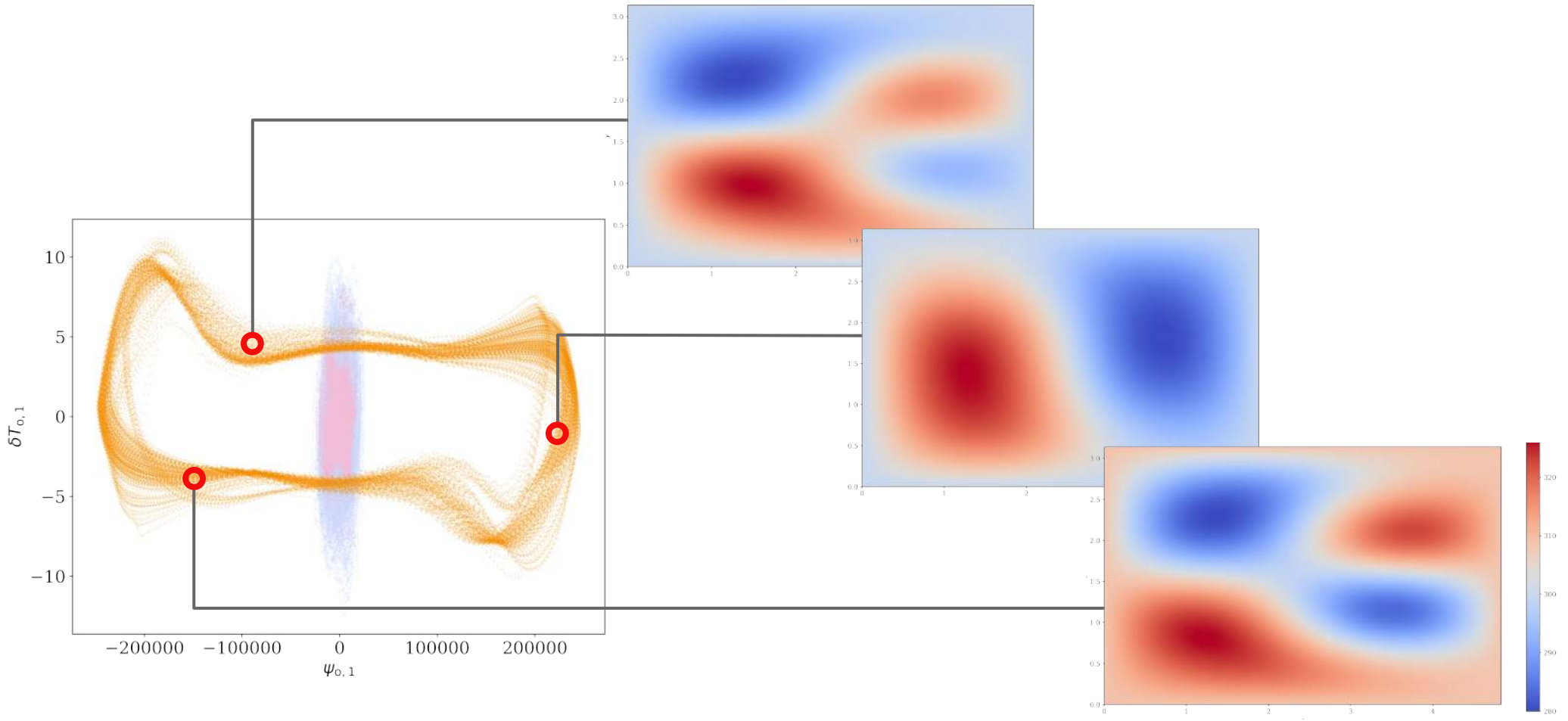


ϕ_2

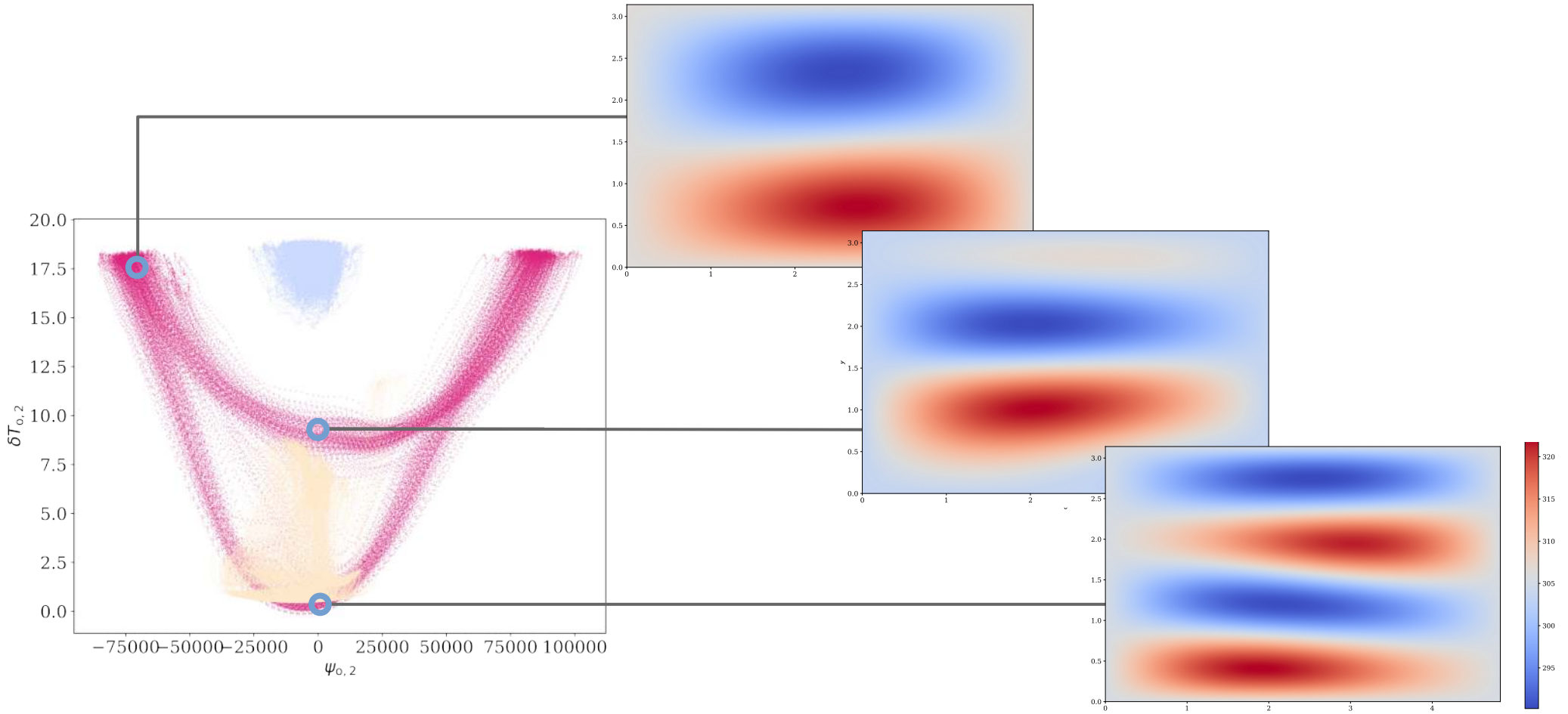




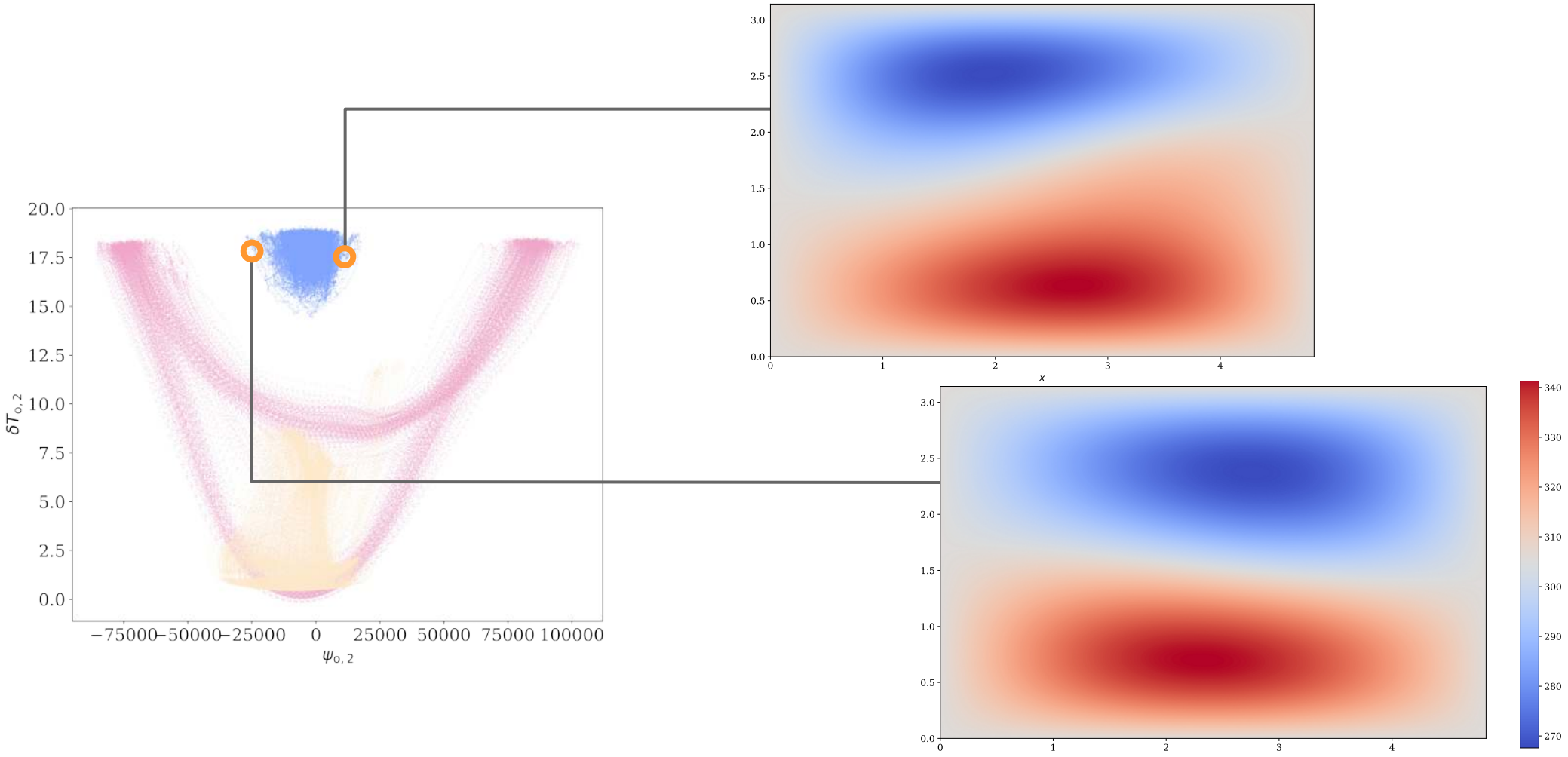
Model Results



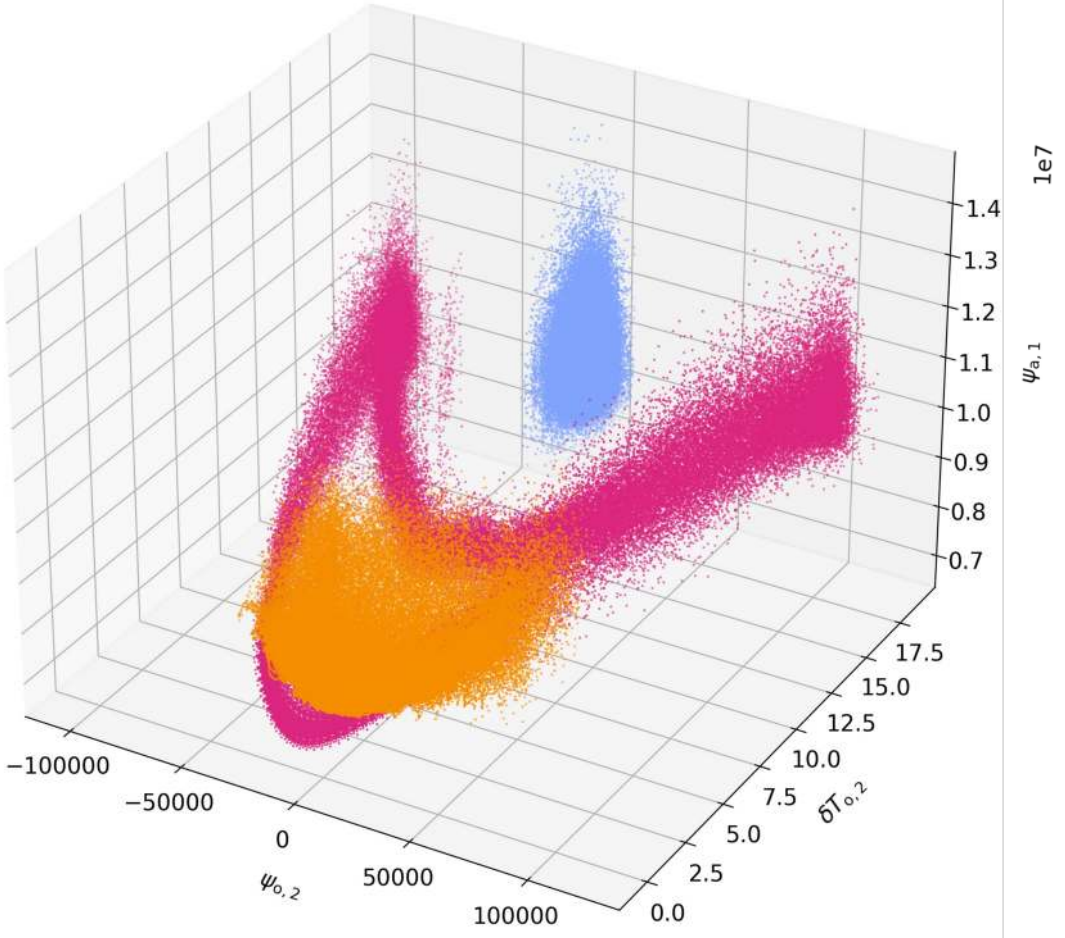
Model Results



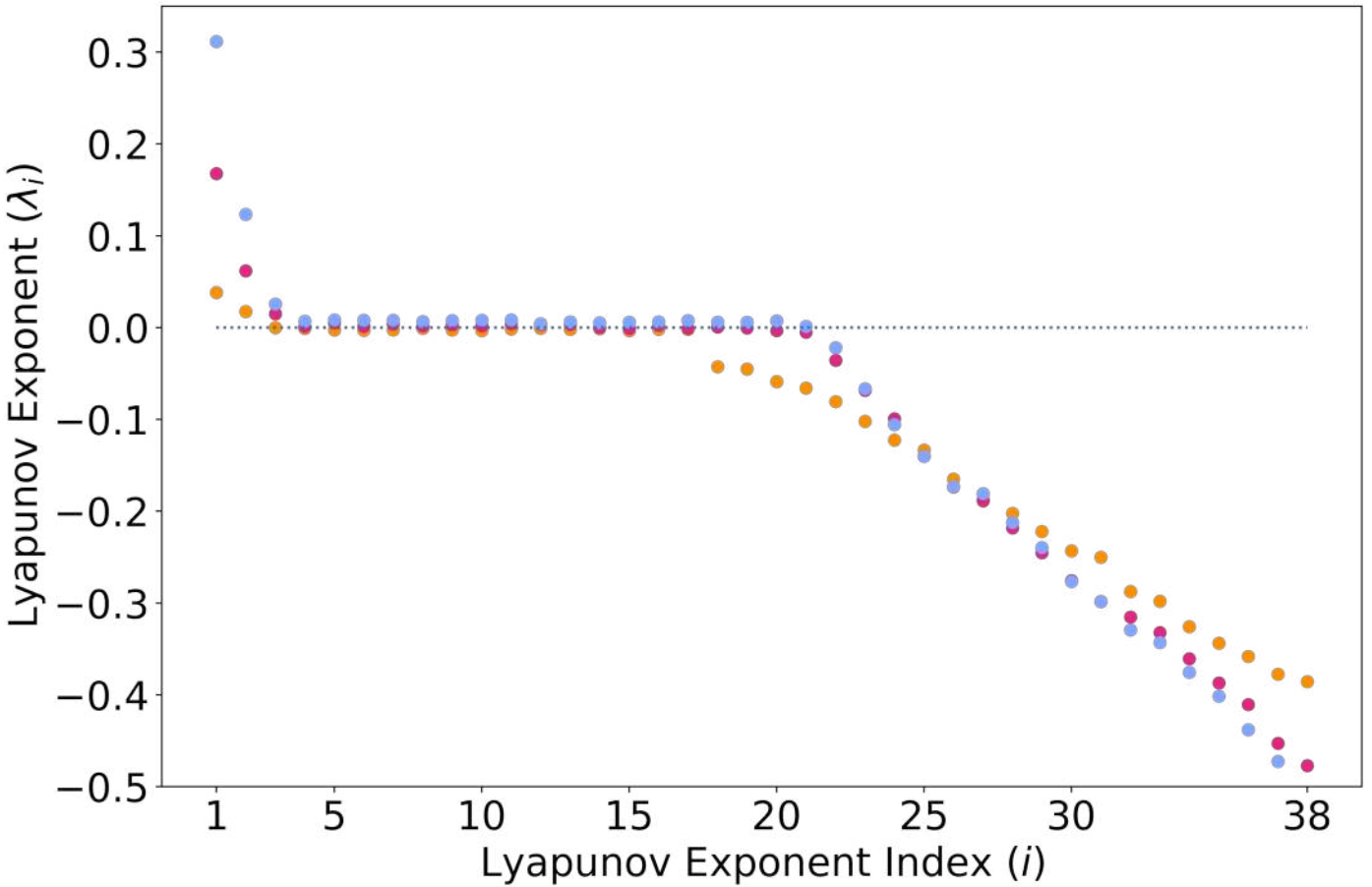
Model Results



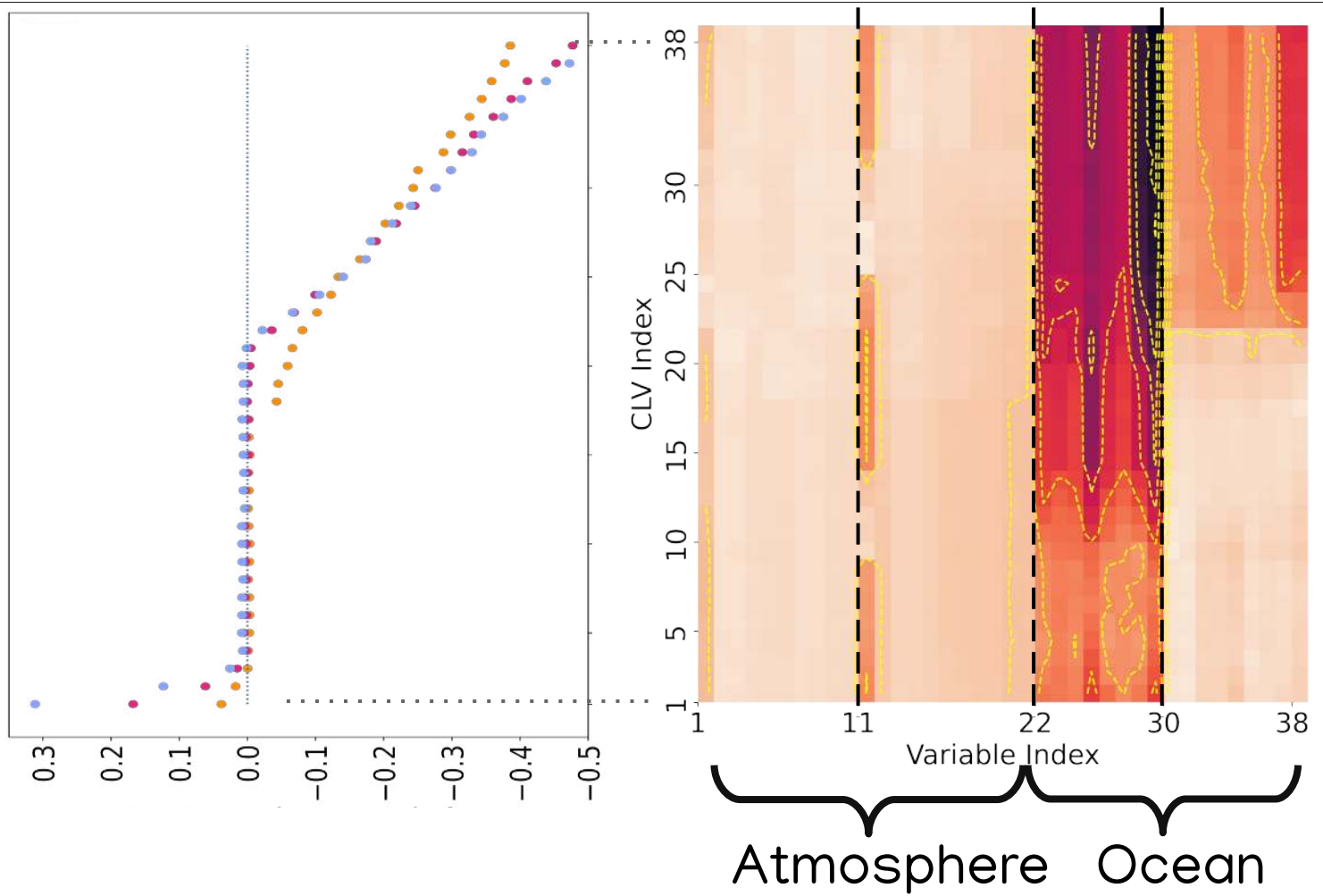
Coupling


1e7
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
$\psi_{a,1}$
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
$\delta T_{o,2}$
−100000
−50000
0
50000
100000
$\psi_{o,2}$

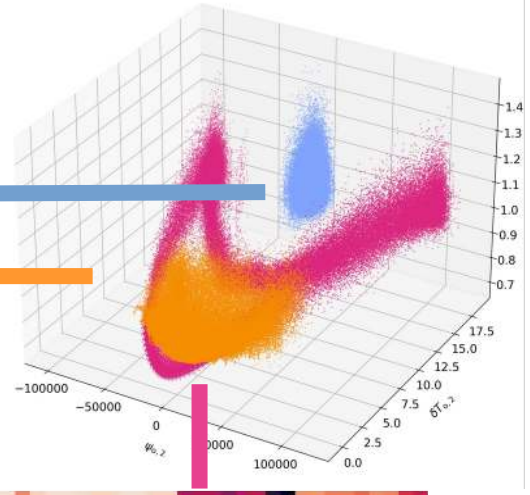
Coupling

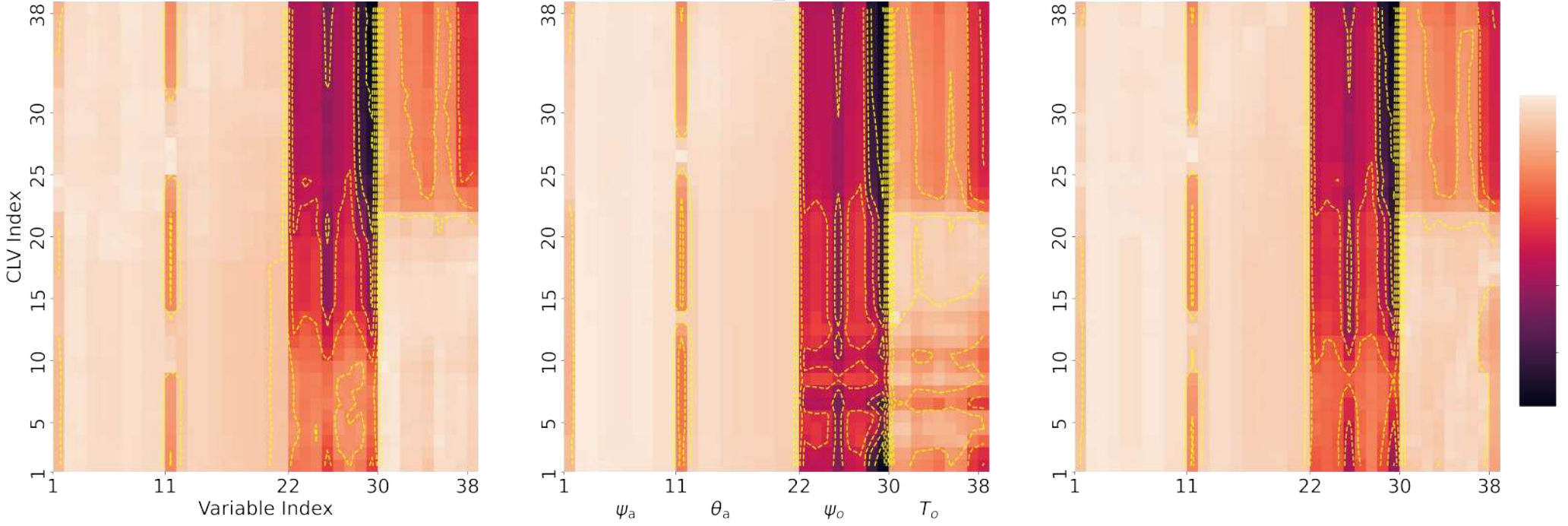

0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
Lyapunov Exponent (λ_i)
1
5
10
15
20
25
30
38
Lyapunov Exponent Index (i)

Coupling

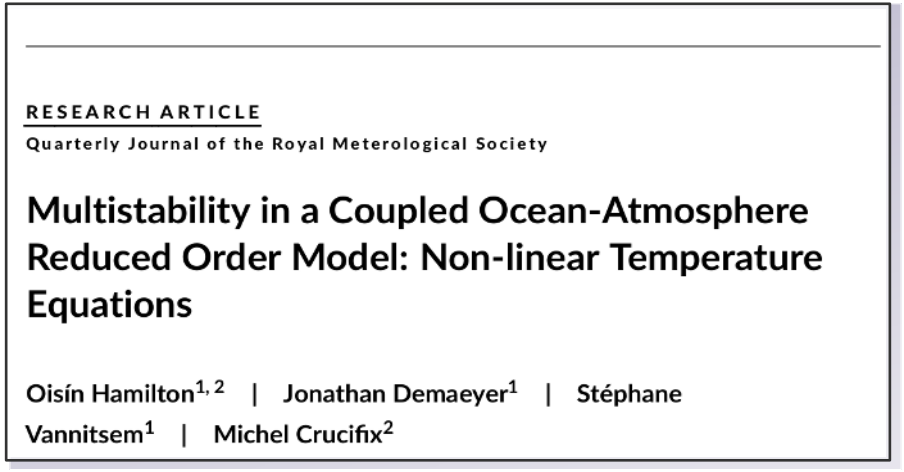

0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
CLV Index
1
5
10
15
20
25
30
38
1
11
22
30
38
Variable Index
Atmosphere
Ocean

Coupling


1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
−100000
−50000
0
100000


CLV Index
1
5
10
15
20
25
30
38
1
11
22
30
38
Variable Index
ψ_a
θ_a
ψ_o
T_o

Additional Information



RESEARCH ARTICLE

Quarterly Journal of the Royal Meterological Society

Multistability in a Coupled Ocean-Atmosphere Reduced Order Model: Non-linear Temperature Equations

Oisín Hamilton[1, 2] | Jonathan Demaeyer[1] | Stéphane Vannitsem[1] | Michel Crucifix[2]

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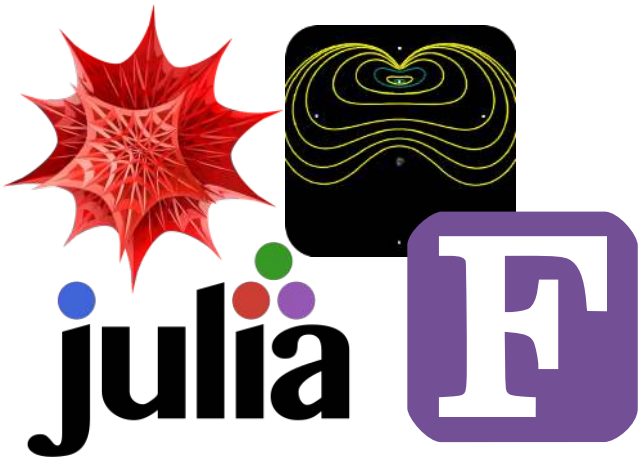

QG Equations

Projected Symbolic ODEs

Conclusions

Not linearising the σT^4 terms produces multiple attractors

These attractors present distinct coupled flows

The coupled flows have different predictability

github.com/Climdyn/qgs

Thank you

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